

DESY 01/003
LU-ITP 01/01
PSI-PR-01-01
UR-1630, ER/40685/965
January 30, 2001
hep-ph/0101257

Off-shell W-pair production — universal versus non-universal corrections

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Electroweak radiative corrections to e^+e^- scattering processes typically amount to $\mathcal{O}(10\%)$ at LEP energies. Their logarithmic increase with energy renders them even more important at future colliders. Although the bulk of these corrections is due to universal process-independent effects, the remaining non-universal corrections are nevertheless phenomenologically important. We describe the structure of the universal corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ in detail and discuss the numerical size of universal and non-universal effects using the Monte Carlo generator RACOONWW[†].

Presented at the

5th International Symposium on Radiative Corrections
(RADCOR–2000)
Carmel CA, USA, 11–15 September, 2000

*Heisenberg fellow of the Deutsche Forschungsgemeinschaft

[†]Program available from <http://www.hep.psi.ch/racoonww/racoonww.html>

1 Introduction

At present, the investigation of W-pair production at LEP2 plays an important role in the verification of the Electroweak Standard Model (SM). Apart from the direct observation of the triple-gauge-boson couplings in $e^+e^- \rightarrow W^+W^-$, the increasing accuracy in the W-pair-production cross-section and W-mass measurements has put this process into the row of SM precision tests [1]. The W-pair cross section is measured at the per-cent level, and the W-boson-mass determination aims at a final accuracy of 30 MeV. Experiments at a future e^+e^- linear collider (LC) with higher luminosity and higher energy will even exceed this precision.

To account for the high experimental accuracy on the theoretical side is a great challenge: the W bosons have to be treated as resonances in the full four-fermion processes $e^+e^- \rightarrow 4f$, and radiative corrections need to be included. While several lowest-order predictions are based on the full set of Feynman diagrams, only very few calculations include radiative corrections beyond the level of universal radiative corrections (see Refs. [2,3] and references therein). These universal corrections comprise the leading process-independent effects; for $e^+e^- \rightarrow WW \rightarrow 4f$ these include universal renormalization effects (running or effective couplings), the Coulomb singularity at the W-pair-production threshold, and initial-state radiation (ISR) in leading-logarithmic approximation. The remaining corrections are usually viewed as non-universal and can only be included by an explicit diagrammatic calculation. In this article we describe the structure of the universal corrections in detail and discuss the size of the non-universal corrections for LEP2 und LC energies. This issue is not only theoretically interesting, it is also important in practice, since many Monte Carlo generators for W-pair production that are in use neglect non-universal electroweak corrections.

The size of non-universal corrections was already estimated by inspecting the pair production of stable W bosons quite some time ago [2,4,5]. For LEP2 energies these effects reduce the total W-pair cross section at the level of 1–2%, but for energies in the TeV range the impact grows to $\mathcal{O}(10\%)$. For differential distributions the size of the non-universal corrections is usually much larger. In the following we investigate the corresponding corrections to off-shell W-pair production, $e^+e^- \rightarrow WW \rightarrow 4f$, by inspecting total cross sections as well as angular and invariant-mass distributions with the Monte Carlo generator RACOONWW [6].

2 Radiative corrections to off-shell W-pair production — state of the art

Fortunately, to match the experimental precision for W-pair production a full one-loop calculation for the four-fermion processes is not needed for most purposes,

in particular for LEP2 physics. Instead it is sufficient to take into account only those radiative corrections that are enhanced by two resonant W bosons. For centre-of-mass (CM) energies E_{CM} not too close to the W-pair-production threshold, say for $E_{\text{CM}} \gtrsim 170 \text{ GeV}$, the neglected $\mathcal{O}(\alpha)$ corrections are of the order $(\alpha/\pi)(\Gamma_W/M_W)$, i.e. below 0.5% even if possible enhancement factors are taken into account. The theoretically clean way to carry out this approximation is the expansion about the two resonance poles, which is called *double-pole approximation* (DPA). A full description of this strategy and of different variants used in the literature (some of them involving further approximations) can be found in Refs. [6–9].

At present, two Monte Carlo programs include $\mathcal{O}(\alpha)$ corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ in DPA and further numerically important higher-order effects: YFSWW3 [7] and RACOONWW [6]. The salient features of the two approaches, which are conceptually very different, as well as detailed comparisons of numerical results are summarized in Ref. [3]. Further numerical results of the two programs can be found in Refs. [10,11]. Both programs have reached an accuracy of roughly $\sim 0.5\%$ for CM energies between 170 GeV and 500 GeV. For higher energies also leading electroweak two-loop effects become important (see also below).

Figure 1 shows a comparison of the results of RACOONWW and YFSWW3 with recent LEP2 data, as given by the LEP Electroweak Working Group [12] for the Summer 2000 conferences. The data are in good agreement with the predictions of the two programs, which differ by about 0.3% at LEP2 energies. Below a CM energy of 170 GeV, the prediction in Figure 1 is continued by GENTLE [13], which does not include the non-universal electroweak corrections. In its new version GENTLE is tuned to reproduce the DPA prediction of RACOONWW and YFSWW3 on the total cross section at LEP2 within a few per mill (see Ref. [3]).

3 Universal electroweak corrections — improved Born approximation

3.1 Preliminaries

Universal radiative corrections are those parts of the full correction that are connected to specific subprocesses, such as collinear photon emission or running couplings, and lead to characteristic enhancement factors. Owing to their universality such corrections are often related to the lowest-order matrix element of the underlying process. In the following we construct an *improved Born approximation* (IBA) for the processes $e^+e^- \rightarrow WW \rightarrow 4f$ that is based on universal corrections only. For the production subprocess the IBA closely follows the approximation formulated in Ref. [4] for on-shell W-pair production. For the W decay the IBA is identical with the lowest-order prediction in the G_μ scheme, as suggested in Ref. [14].

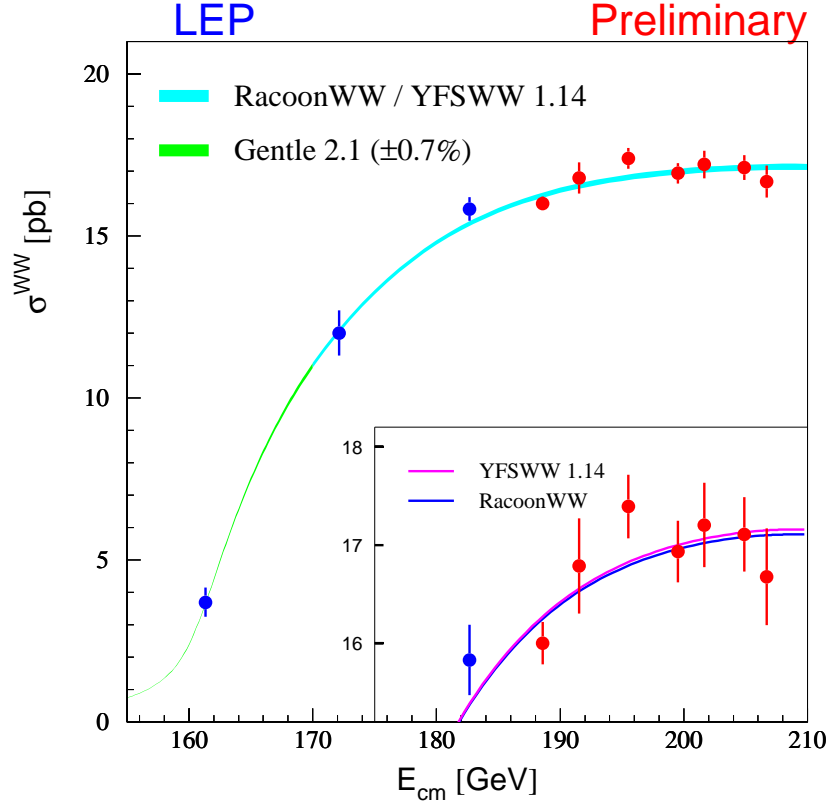


Figure 1: Total WW production cross section at LEP2 as given by the LEPEWWG [12]

In order to define the IBA, we first need the lowest-order matrix element of the process

$$\begin{aligned}
 e^+(p_+, \sigma_+) + e^-(p_-, \sigma_-) &\rightarrow W^+(k_+, \lambda_+) + W^-(k_-, \lambda_-) \\
 &\rightarrow f_1(k_1, \sigma_1) + \bar{f}_2(k_2, \sigma_2) + f_3(k_3, \sigma_3) + \bar{f}_4(k_4, \sigma_4). \quad (3.1)
 \end{aligned}$$

The arguments label the momenta p_{\pm} , k_i and helicities $\sigma_i = \pm 1/2$, $\lambda_j = 0, \pm 1$ of the corresponding particles. The cross section that is defined by including only the so-called *signal diagrams* for W-pair-mediated four-fermion production, which are shown in Figure 2, is called CC03 cross section¹. Note that the masses of the external fermions (not the ones in closed fermion loops) are neglected whenever possible. In the absence of photon radiation this, in particular, implies that we have helicity conservation for the initial e^+e^- system, i.e. only the combination $\sigma_- = -\sigma_+$ con-

¹Of course, the CC03 cross section is a non gauge-invariant quantity. However, evaluated in the 't Hooft–Feynman gauge it approximates the full cross sections for W-pair-mediated $4f$ production very well, as long as no electrons or positrons are in the final state. Therefore, the CC03 cross section is widely used in the literature (see also Refs. [2,3]).

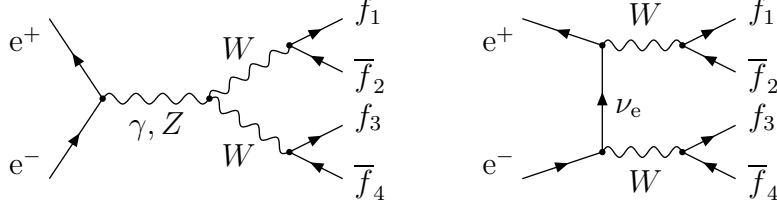


Figure 2: Lowest-order signal diagrams for $e^+e^- \rightarrow WW \rightarrow 4f$

tributes, and we can define $\sigma = \sigma_- = -\sigma_+$. For definite electron helicity σ , the lowest-order CC03 matrix element is given by

$$\mathcal{M}_{\text{Born,CC03}}^{e^+e^- \rightarrow WW \rightarrow 4f, \sigma}(p_+, p_-, k_+, k_-, k_+^2, k_-^2) = \sum_{n=1}^3 F_{n, \text{Born}}^\sigma(s, t) \mathcal{M}_n^\sigma(p_+, p_-, k_+, k_-, k_+^2, k_-^2), \quad (3.2)$$

where \mathcal{M}_n^σ are so-called standard matrix elements (SME) containing the spinor chains of the external fermions, and $F_{n, \text{Born}}^\sigma(s, t)$ are invariant functions containing couplings and propagator factors. In lowest order only three SME and invariant functions contribute. Following the notation and conventions of Ref. [6], these read ($\omega_\pm = (1 \pm \gamma_5)/2$)

$$\begin{aligned} \mathcal{M}_1^\sigma &= \bar{v}(p_+) \not{\epsilon}_+^* (\not{k}_+ - \not{p}_+) \not{\epsilon}_-^* \omega_\sigma u(p_-), \\ \mathcal{M}_2^\sigma &= \bar{v}(p_+) \frac{1}{2} (\not{k}_+ - \not{k}_-) \omega_\sigma u(p_-) (\epsilon_+^* \epsilon_-^*), \\ \mathcal{M}_3^\sigma &= \bar{v}(p_+) \not{\epsilon}_+^* \omega_\sigma u(p_-) (\epsilon_-^* k_+) - \bar{v}(p_+) \not{\epsilon}_-^* \omega_\sigma u(p_-) (\epsilon_+^* k_-) \end{aligned} \quad (3.3)$$

with “effective W-polarization vectors”

$$\begin{aligned} \epsilon_+^{*, \mu} &= \frac{e}{\sqrt{2}s_w} \frac{1}{k_+^2 - M_W^2 + iM_W \Gamma_W} \bar{u}(k_1) \gamma^\mu \omega_- v(k_2), \\ \epsilon_-^{*, \mu} &= \frac{e}{\sqrt{2}s_w} \frac{1}{k_-^2 - M_W^2 + iM_W \Gamma_W} \bar{u}(k_3) \gamma^\mu \omega_- v(k_4), \end{aligned} \quad (3.4)$$

and

$$\begin{aligned} F_{1, \text{Born}}^\sigma(s, t) &= \frac{e^2}{2s_w^2 t} \delta_{\sigma-}, \\ F_{3, \text{Born}}^\sigma(s, t) &= -F_{2, \text{Born}}^\sigma(s, t) = \frac{2e^2}{s} - \frac{2e^2}{s - M_Z^2} \left(1 - \frac{\delta_{\sigma-}}{2s_w^2}\right). \end{aligned} \quad (3.5)$$

The actual values of the input parameters e , M_W , M_Z , and s_w depend on the input-parameter scheme. In the G_μ -scheme the electromagnetic coupling e is deduced from the Fermi constant G_μ using the tree-level relation $e^2 = 4\sqrt{2}G_\mu M_W^2 s_w^2$, and the weak

mixing angle is fixed by the gauge-boson masses, which are independent input parameters, $s_w^2 = 1 - M_W^2/M_Z^2$.

Before we define the IBA we comment on the calculation of the full factorizable one-loop correction in DPA², which is described in Ref. [6], and its relation to the decomposition (3.2). In this case six independent SME contribute for each value of σ , and the functions F_n^σ contain standard loop integrals. Moreover, in order to guarantee the gauge invariance of the corrections, which is mandatory for consistency, it is necessary to perform an on-shell projection of the external fermion momenta k_i . This means that the k_i are changed to related momenta \hat{k}_i in such a way that $\hat{k}_\pm^2 = M_W^2$. The off-shell values k_\pm^2 are kept only in the propagator factors of (3.4).³

3.2 Improved Born approximation

The first step in the construction of the IBA consists in a modification of the Born matrix element in such a way that the universal renormalization effects induced by the running of α and by $\Delta\rho$ are absorbed. This is achieved [4] by the replacements

$$\frac{e^2}{s_w^2} \rightarrow 4\sqrt{2}G_\mu M_W^2, \quad e^2 \rightarrow 4\pi\alpha(s) \quad (3.6)$$

in the lowest-order functions $F_{i,\text{Born}}^\sigma$ of (3.5), which implies that weak-isospin exchange involves the coupling $G_\mu M_W^2$ and pure photon exchange the coupling $\alpha(s)$. The running of the electromagnetic coupling is induced by light (massless) charged fermions only, i.e. we evaluate $\alpha(s)$ by

$$\alpha(s) = \frac{\alpha(M_Z^2)}{1 - \frac{\alpha(M_Z^2)}{3\pi} \ln(s/M_Z^2) \sum_{f \neq t} N_f^c Q_f^2} \quad (3.7)$$

with the value $\alpha(M_Z^2) = 1/128.887$ taken from the fit [15] of the hadronic vacuum polarization to the empirical ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. Thus, the basic matrix element for the IBA reads

$$\mathcal{M}_{\text{IBA}}^{e^+e^- \rightarrow WW \rightarrow 4f, \sigma}(p_+, p_-, k_+, k_-, k_+^2, k_-^2) = \sum_{n=1}^3 F_{n,\text{IBA}}^\sigma(s, t) \mathcal{M}_n^\sigma(p_+, p_-, k_+, k_-, k_+^2, k_-^2) \quad (3.8)$$

²In DPA the virtual one-loop correction consists of *factorizable* and *non-factorizable* contributions. The factorizable corrections are the ones that are related to the W-pair-production and W-decay subprocesses. The non-factorizable corrections comprise the remaining doubly-resonant virtual corrections and include all diagrams with photon exchange between the production and decay subprocesses.

³This on-shell projection also renders the CC03 cross section gauge-invariant, leading to the so-called DPA Born cross section. However, the DPA Born cross section is a much worse approximation for the $4f$ cross section than the CC03 variant (see also Refs. [3,8]).

with

$$\begin{aligned}
F_{1,\text{IBA}}^\sigma(s, t) &= \frac{2\sqrt{2}G_\mu M_W^2}{t} \delta_{\sigma-}, \\
F_{3,\text{IBA}}^\sigma(s, t) &= -F_{2,\text{IBA}}^\sigma(s, t) = \frac{4\sqrt{2}G_\mu M_W^2}{s - \hat{M}_Z^2} \delta_{\sigma-} - \frac{8\pi\alpha(s)\hat{M}_Z^2}{s(s - \hat{M}_Z^2)}.
\end{aligned} \tag{3.9}$$

Note that we have used the complex Z-boson mass $\hat{M}_Z^2 = M_Z^2 - iM_W\Gamma_Z$ in order to regularize the Z resonance below the W-pair-production threshold; otherwise the ISR convolution over the reduced CM energy would lead to complications (see below).

Another important virtual correction is induced by the Coulomb singularity near the W-pair-production threshold. We include this effect in the calculation of the “hard” IBA cross section $\hat{\sigma}_{\text{IBA}}^{e^+e^- \rightarrow \text{WW} \rightarrow 4f}$,

$$\int d\hat{\sigma}_{\text{IBA}}^{e^+e^- \rightarrow \text{WW} \rightarrow 4f}(p_+, p_-) = \frac{1}{2s} \int d\Phi_{4f} |\mathcal{M}_{\text{IBA}}^{e^+e^- \rightarrow \text{WW} \rightarrow 4f}|^2 \left[1 + \delta_{\text{Coul}}(s, k_+^2, k_-^2) g(\bar{\beta}) \right], \tag{3.10}$$

where the correction factor δ_{Coul} is given by [16,17]

$$\begin{aligned}
\delta_{\text{Coul}}(s, k_+^2, k_-^2) &= \frac{\alpha(0)}{\bar{\beta}} \text{Im} \left\{ \ln \left(\frac{\beta - \bar{\beta} + \Delta_M}{\beta + \bar{\beta} + \Delta_M} \right) \right\}, \\
\bar{\beta} &= \frac{\sqrt{s^2 + k_+^4 + k_-^4 - 2sk_+^2 - 2sk_-^2 - 2k_+^2 k_-^2}}{s}, \\
\beta &= \sqrt{1 - \frac{4(M_W^2 - iM_W\Gamma_W)}{s}}, \quad \Delta_M = \frac{|k_+^2 - k_-^2|}{s}
\end{aligned} \tag{3.11}$$

with the fine-structure constant $\alpha(0)$. The auxiliary function

$$g(\bar{\beta}) = \left(1 - \bar{\beta}^2 \right)^2 \tag{3.12}$$

restricts the impact of δ_{Coul} to the threshold region where it is valid. Its actual form (and its occurrence) is somewhat ad hoc but justified by a numerical comparison to the full $\mathcal{O}(\alpha)$ correction. Omitting this factor would lead to a constant positive correction of a few per mill above threshold, although the correct non-universal correction is even negative.

The last ingredient in the IBA is the leading-logarithmic contribution induced by initial-state radiation (ISR). We follow the structure-function approach [18], where the full IBA cross section σ_{IBA} reads

$$\int d\sigma_{\text{IBA}} = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{\text{ee}}^{\text{LL}}(x_1, Q^2) \Gamma_{\text{ee}}^{\text{LL}}(x_2, Q^2) \int d\hat{\sigma}_{\text{IBA}}^{e^+e^- \rightarrow \text{WW} \rightarrow 4f}(x_1 p_+, x_2 p_-). \tag{3.13}$$

The structure functions $\Gamma_{\text{ee}}^{\text{LL}}(x, Q^2)$ include the leading logarithms $[\alpha \ln(Q^2/m_e^2)]^n$ up to order $n = 3$, and the soft-photon effects are exponentiated; the explicit expressions can also be found in Refs. [2,6]. The QED splitting scale Q^2 is not fixed in leading-logarithmic approximation and has to be set to a typical momentum scale of the process. It can be used to adjust the IBA to the full correction, but also to estimate the intrinsic uncertainty of the IBA by choosing different values for Q^2 .

Finally, we have to fix the W-boson width Γ_W in the evaluation of the IBA. In order to avoid any kind of mismatch with the decay, Γ_W should be calculated in lowest order using the G_μ scheme. This choice guarantees that the “effective branching ratios”, which result after integrating out the decay parts, add up to one when summing over all channels. Of course, if naive QCD corrections are taken into account by multiplying with $(1 + \alpha_s/\pi)$ for each hadronically decaying W boson, these QCD factors also have to be included in the calculation of the total W width.

Note that unlike the full one-loop calculation in DPA, the IBA is also applicable near the W-pair production threshold, since no pole expansion is involved.

4 Comparison of the improved Born approximation with state-of-the-art results

4.1 Total cross section

In order to investigate the reliability of the IBA defined in (3.13), we have implemented this IBA in the Monte Carlo program RACOONWW, which provides state-of-the-art predictions for the full $\mathcal{O}(\alpha)$ corrections in DPA, as discussed above. For the following numerical evaluations we have adopted the input-parameter set of Refs. [3,6].

Figure 3 compares different predictions for the total cross section (without any phase-space cuts) for the semileptonic process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu(\gamma)$ for CM energies E_{CM} up to 1 TeV. The IBA is evaluated for the two different scales $Q^2 = s = E_{\text{CM}}^2$ and $Q^2 = |t_{\text{min}}| = E_{\text{CM}}^2 \left(1 - \sqrt{1 - 4M_W^2/E_{\text{CM}}^2}\right)/2 - M_W^2$, and “best” labels the RACOONWW prediction including all universal and non-universal corrections as described in detail in Ref. [6]. The motivation for $Q^2 = s$ is obvious; $Q^2 = |t_{\text{min}}|$ is motivated by the fact that t_{min} corresponds to the minimal momentum transfer in the t -channel diagrams for forward scattering of on-shell W bosons, which dominates the cross section. The comparison of the corresponding relative corrections (normalized to the CC03 Born cross section in G_μ scheme) is shown in Figure 4.

For LEP2 energies, i.e. energies below 210 GeV, the difference between the two IBA versions reflects the typical uncertainty of 1–2% inherent in all predictions that neglect non-universal electroweak corrections. It turns out that the IBA with $Q^2 = s$ is closer to the “best” prediction, with a maximal deviation at the upper LEP2

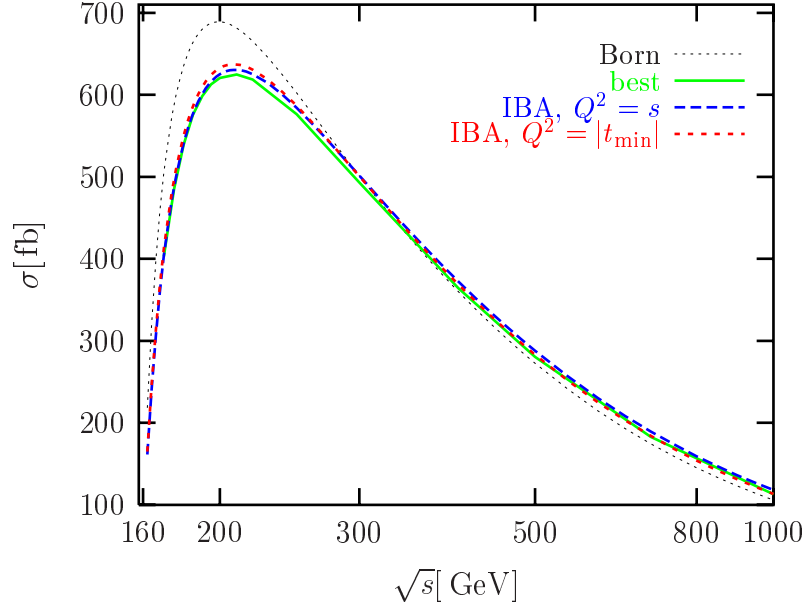


Figure 3: Predictions for the total cross section for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ based on various approximations for radiative corrections

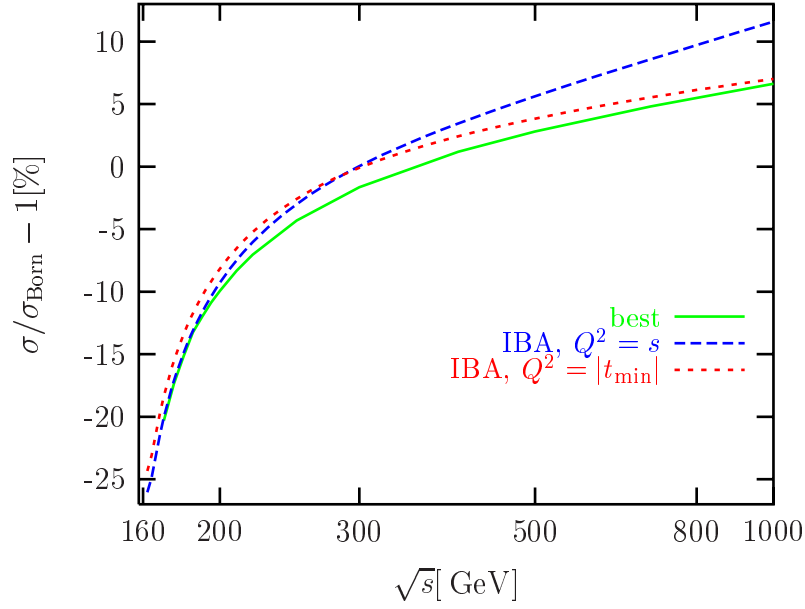


Figure 4: Relative corrections to the total cross section for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ in various approximations

energies: $\sim 0.6\%$ at 200 GeV and $\sim 0.8\%$ at 210 GeV. Note that the “best” prediction is not included below 170 GeV, since the uncertainty of all predictions based on a DPA formally runs out of control near the W-pair-production threshold. On the other hand, the IBA does not suffer from this constraint. Since the IBA with $Q^2 = s$ agrees with the “best” prediction near 170 GeV at the per-mill level this IBA version is an appropriate extrapolation of the “best” RACOONWW prediction down to the W-pair-production threshold. Of course, the theoretical uncertainty below 170 GeV is then of the order of one to a few per cent.

For LC energies the IBA becomes more and more uncertain; for 1 TeV the two IBA versions differ already by $\sim 5\%$. This signals that non-leading electroweak corrections become more and more important. The dominant effects are due to Sudakov logarithms [19] of the type $\alpha \ln^2(s/M_W^2)$ which originate from the exchange of soft and collinear massive gauge bosons, i.e. W and Z bosons. The IBA does not account for these effects. Nevertheless the IBA with $Q^2 = |t_{\min}|$ follows the “best” prediction within $\sim 1\text{--}2\%$ even for high energies. This is plausible, because the total cross section is strongly dominated by the t -channel pole for forward scattering for high energies, and this contribution is well approximated by the IBA. Note, however, that the good agreement could not be predicted without a comparison with the full DPA correction including non-universal electroweak corrections. On the other hand, it can be expected that the quality of the IBA with $Q^2 = |t_{\min}|$ also becomes worse if forward scattering is excluded or suppressed by phase-space cuts; this issue is further discussed below in the context of differential distributions.

4.2 Differential distributions

In order to define differential distributions, the kinematic information on the fermion momenta in $e^+e^- \rightarrow WW \rightarrow 4f(\gamma)$ is required. In the presence of photon radiation, a consistent treatment of photons that are soft or collinear to charged fermions is crucial. If such photons are not recombined with the nearest charged fermion, i.e. if these photon-fermion systems are not treated as single “quasi-particles”, the bare fermion momenta in general lead to distributions that are not IR-safe, i.e. they involve mass-singular logarithms of the form $\alpha \ln m_f$. For fermions other than muons such effects are definitely unphysical. In order to avoid such artifacts, we recombine soft and collinear photons according to the procedure⁴ described in Refs. [3,6].

⁴In this approach, first photons close to the beams are dropped in events, i.e. their momenta are set to zero. If the photon survives the cut to the beam, it is recombined with the charged fermion f if $M_{f\gamma} < M_{\text{rec}}$, where f is the fermion with the smallest invariant mass $M_{f\gamma}^2 = (p_f + k_\gamma)^2$ with the photon. Finally, events are discarded in which charged fermions are close to the beam. The size of the recombination cut M_{rec} , thus, determines how many photons are recombined with the charged fermions. In Refs. [3,6] the two values $M_{\text{rec}} = 5$ GeV and 25 GeV are chosen, defining a “bare” and a “calo(rimetric)” setup, respectively.

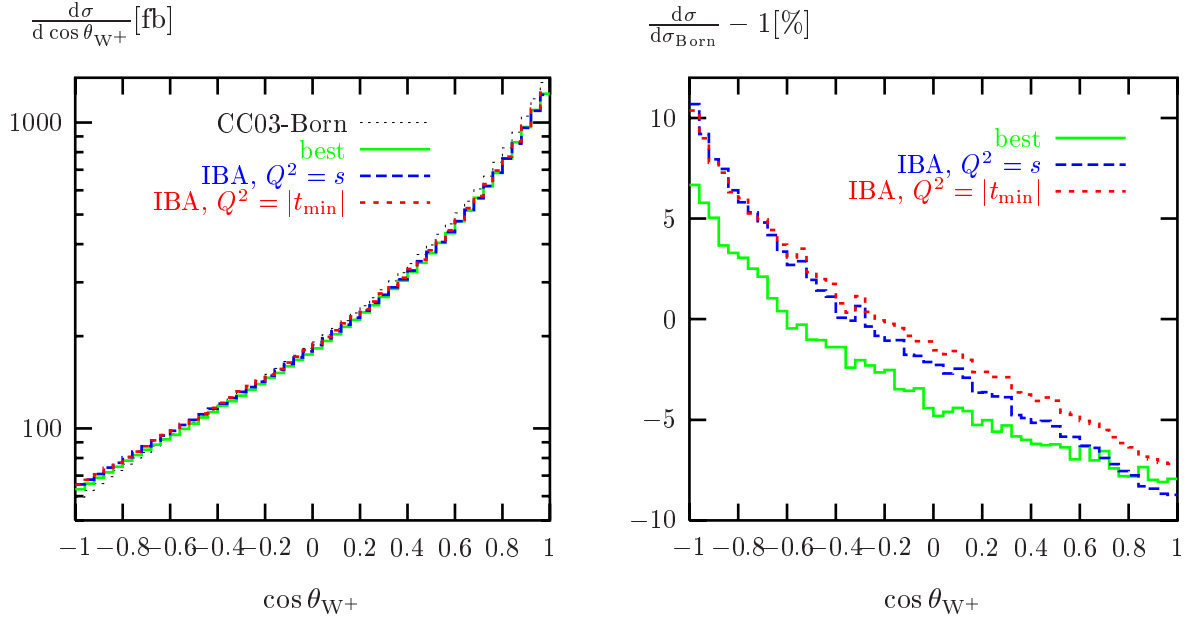


Figure 5: Predictions for the W^+ -production-angle distribution (left) and corresponding relative corrections (right) for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at $E_{\text{CM}} = 200 \text{ GeV}$ based on various approximations for radiative corrections

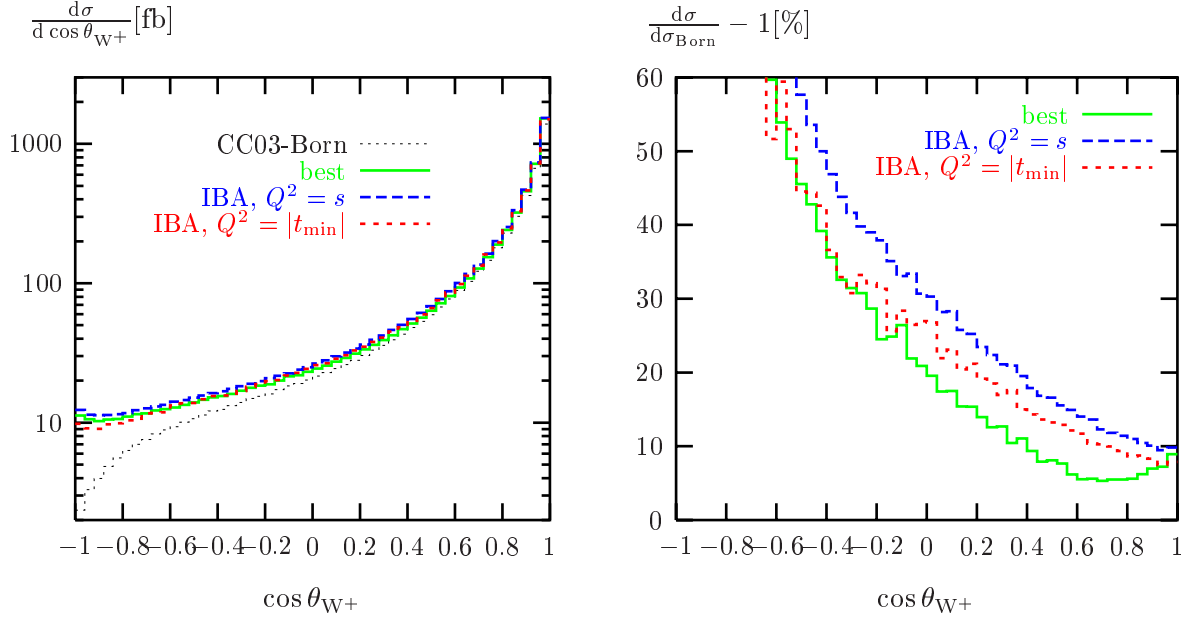


Figure 6: Predictions for the W^+ -production-angle distribution (left) and corresponding relative corrections (right) for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at $E_{\text{CM}} = 500 \text{ GeV}$ based on various approximations for radiative corrections

In Figures 5 and 6 the full RACOONWW and IBA predictions for the W^+ -production-angle distribution are compared for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at the typical LEP2 energy of $E_{\text{CM}} = 200$ GeV and the LC energy $E_{\text{CM}} = 500$ GeV. The uncertainty of the IBA predictions induced by the QED splitting scale Q is about 1–2% and $\sim 5\%$ for LEP and LC energies, respectively. The deviation of the IBA prediction from the full result is up to $\sim 5\%$ and ~ 5 –10%, where the agreement is best for forward scattering ($\cos\theta_{W^+} \rightarrow 1$), as anticipated above. The IBA uncertainty and the deviation from the full result further grow with increasing energy. This comparison is performed using the “calo” setup for the photon recombination of photons, but the sensitivity of the W -production-angle distribution to the recombination procedure is very weak (see Refs. [3,6]).

The sensitivity to photon recombination is maximal in the invariant-mass distributions of the reconstructed W bosons, which can be seen by comparing Figures 7 and 8. The full correction shows a very strong dependence on the recombination procedure, which was discussed in Ref. [10] in detail. The more inclusive recombination (“calo”) leads to large positive corrections above resonance and thus to a shift of the resonance to the right, which can be of the order of some 10 MeV [10]. Since this distortion of the W line shape is mainly induced by final-state radiation and radiation off the W bosons, the IBA, as defined above, does not account for this effect. It is obvious that the W -invariant-mass distributions can only be properly described if photon radiation from the W -decay processes is taken into account properly. Figures 7 and 8 refer to the LEP energy $E_{\text{CM}} = 200$ GeV, but this conclusion is, of course, valid for all energies.

5 Conclusions

Electroweak radiative corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ typically amount to $\mathcal{O}(10\%)$ at LEP2 energies and further increase for higher energies. We have explicitly given analytical results for the universal process-independent corrections, which include effective coupling constants, the Coulomb singularity near the W -pair-production threshold, and leading ISR effects. They have been implemented in the Monte Carlo generator RACOONWW, which calculates the full $\mathcal{O}(\alpha)$ corrections in double-pole approximation. Using this program a comparison between universal effects and the full correction has been presented.

For LEP2 energies the universal corrections are dominant, and the remaining non-universal contributions reduce the total W -pair cross section by 1–2%. In angular distributions non-universal effects can reach several per cent, mainly in regions where the cross section is small. The radiative corrections to W -invariant-mass distributions lead to a distortion of the W resonance, which is mainly due to photon radiation off

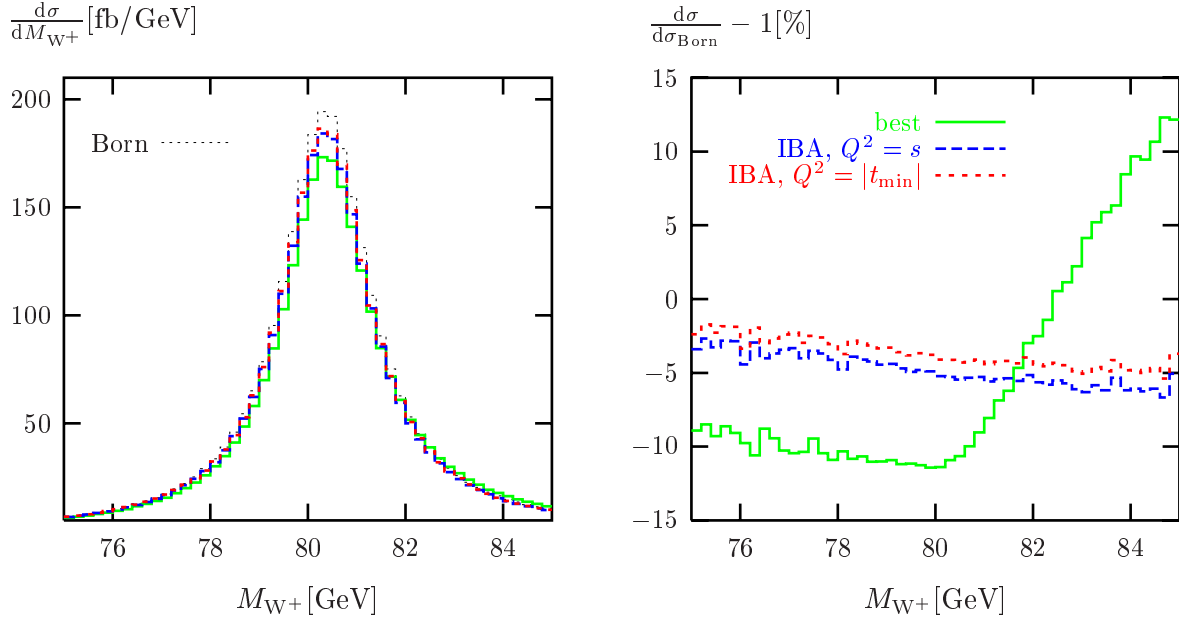


Figure 7: Predictions for the W^+ -invariant-mass distribution (left) and corresponding relative corrections (right) for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at $E_{\text{CM}} = 200$ GeV based on various approximations for radiative corrections, using the “calo” setup for photon recombination

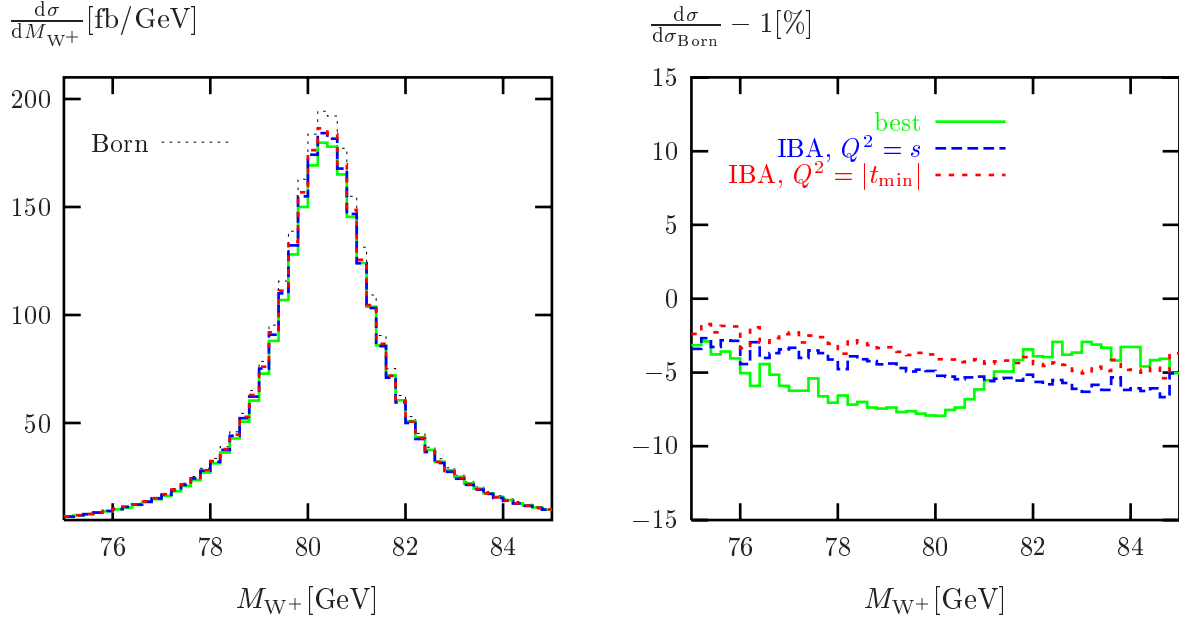


Figure 8: Predictions for the W^+ -invariant-mass distribution (left) and corresponding relative corrections (right) for the process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at $E_{\text{CM}} = 200$ GeV based on various approximations for radiative corrections, using the “bare” setup for photon recombination

the charged final-state fermions and off the W bosons. This line-shape distortion is not accounted for by the above-mentioned universal effects.

For LC energies, i.e. energies up to the TeV range, non-universal effects become more and more important. While the universal effects still describe W-pair production in the forward region within some per cent, non-universal corrections reach the order of several 10% for intermediate and large W-production angles.

Acknowledgments

We would like to thank the organizers of the conference, in particular Howard Haber, for providing a very pleasant atmosphere during the whole conference. Two of us (A.D. and S.D.) would like to thank the organizers for financial support. This work was supported in part by the Swiss Bundesamt für Bildung und Wissenschaft, by the European Union under contract HPRN-CT-2000-00149, by the U.S. Department of Energy under grant DE-FG02-91ER40685 and by the U.S. National Science Foundation under grant PHY-9600155.

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